Geometry 2015-16
Final group exploration problems (v1)

Names ____ Period __

The central question we will explore today is: **What is the center of a triangle?**

Part 1: Circles

Here are four conjectures I claim are **equivalent**:

1. Every triangle has a circle that passes through all of its vertices.
2. For any three non-collinear points, there is a circle that passes through them.
3. For any three non-collinear points, there is a fourth point that is equidistant from all three.
4. The perpendicular bisectors of a triangle's three sides are concurrent; the point of concurrency is called the triangle's **circumcenter**.

Your first task is to figure out, and explain, **why these four conjectures are equivalent**.

Next, prove the fourth conjecture. Include a diagram. Here is the setup:

Given: ΔABC.
Lines *m* and *n* are the perpendicular bisectors of sides AB and BC; *m* and *n* intersect at point P.

Prove: The perpendicular bisector of side AC passes through P.

Part 2: Medians

A median of a triangle is defined as a line segment from a vertex of the triangle to the opposite side's midpoint.

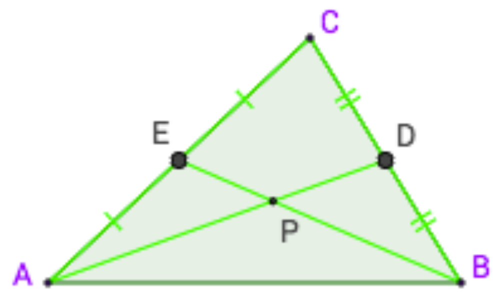
First, have everyone in the group draw their own ΔABC. In a different color, make the three medians.

Next, **prove the following statement** by **answering the questions**, **adding to the diagram**, and **filling the blanks** below.

Theorem. The medians of a triangle are concurrent.

Given: AD and BE are medians of ΔABC, and intersect at point P.

Prove: P lies on the third median of ΔABC.



Proof:

Draw line CP. What permits this?

Choose point R on CP such that CP = PR. What permits this?

Draw segments AR and RB. What permits this?

EB // AR (or, equivalently, EP // AR and PB // AR), and similarly, AD (or AP or PB) is parallel to RB. Why?

APBR is a ______________________________. Why?

PR bisects AB, because ____.

Let Q be the intersection of PR and AB. Q is the midpoint of AB by definition.

Therefore, CQ is the third median of ΔABC, by ____, and P lies on CQ, Q.E.D.

Now that we've proven the theorem, we can take the following definition.

Definition. The centroid of a triangle is the point where the three medians are concurrent.

Part 3: Discussion

Discuss and **explore** the following questions with your group.

We’ve seen two possible answers to our question of **what is the center of a triangle**.
What are the pros and cons of calling each of these two points — the **circumcenter** and the **centroid** — the “center”?

Is one of them clearly a better center than the other? Why, or why not?

How can each of the two centers be constructed? Is one easier to construct than the other?

Are there any circumstances under which you expect the two centers would be the same point?

Do you notice any other interesting features of either center, or of the constructions you did?